

Relativistic Deduction of a Stationary Tohu-va-Bohu Background Cosmology

Peter Ostermann

Given there has been something where the big-bang origin of our evolutionary cosmos took place: What is the relativistic line element describing the energy density and pressure of such a pre-existing universal background ('tohu va bohu')?

Two simple postulates are used to deduce the one and only stationary solution of general relativity [1] implying a constant universal speed of light as well as redshift parameters independent of time (thus in contradiction to the so-called 'Steady-state'-Theory). It is shown that its gravitational 'dark' pressure, corresponding to a stationarily changing cosmological constant, must be negative.

There is a struggle of local SRT (standing for quantum mechanics) and universal GRT (standing for gravitation). In particular, intrinsic limitations of proper length and time are derived, which would cause a stationary background universe to be chaotic.

The model of a stationary background universe

Two postulates are sufficient to deduce the relativistic line element of a stationary universe (SUM) :

Postulate I - The universe is stationary, homogeneous, and isotropic, though on scales large enough only.

Postulate II - Except for deviations caused by local inhomogeneities the universal speed of light is constant.

The solution is

$$d\sigma_{\text{SUM}}^{*2} = \zeta_{\text{SUM}}^{*2} \left\{ c^2 dt^{*2} - dl^{*2} \right\} = \zeta_{\text{SUM}}^{*2} d\sigma_{\text{SRT}}^{*2} \quad (1)$$

where

$$\zeta_{\text{SUM}}^* = e^{Ht^*}. \quad (2)$$

An asterisk '*' always means *universal coordinates* (i.e. 'conformal' t^* and 'comoving' \vec{r}^*).

Because of the exponential time scalar e^{Ht^*} , all relative temporal changes depend on *differences* $\Delta t^* = t^* - t_0^*$ solely. No special point t_0^* of the universal time scale is preferred.

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According to (1), (2) atomic clocks at rest with respect to the Cosmic Microwave Background (CMB) show local proper time and local proper length

$$\begin{aligned} dt_{\text{SRT}} &\approx e^{Ht^*} dt^*, \\ dl_{\text{SRT}} &\approx e^{Ht^*} dl^*. \end{aligned} \quad (3)$$

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Taking into account the universal light time $\Delta t^{*} = l^{*}/c$, the usual definition $z \equiv \lambda_{\text{observed}}/\lambda_{\text{emitted}} - 1$ leads to redshift parameters independent of time:

$$z = e^{Hl^{*}/c} - 1 \Leftrightarrow l^{*} = \frac{c}{H} \ln(1+z) \quad (4)$$

This applies to galaxies statistically at rest with respect to the CMB, i.e. $l^{*} = \text{constant}$. This means in addition to local 'proper' length l_{SRT} , the quantity l^{*} is a real physical but universal distance measure since it is actually an immediate measurand by time-independent values of z .

The energy-stress tensor given by Einstein's equations may be written in the form

$$E_{ik}^* = \begin{pmatrix} \frac{2H^2}{c^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} - \kappa p^* g_{ik} = \kappa T_{ik}^* \quad (5)$$

demanding a negative 'dark' gravitational pressure

$$p^* = -\frac{\rho^*}{3} \quad (6)$$

where $\rho^* \equiv T_0^0 = \rho_c e^{-2Ht^*}$ is the full energy density. Obviously p^* corresponds to a stationarily changing cosmological 'constant'. To state it explicitly, this gravitational pressure *must* be negative because the walls of a large-scale box including a plenty of galaxies at rest ('comoving' coordinates), would have to pull outwards, if those inside should not mass together after those outside had been removed.

With respect to (5) the phenomenological density of matter is only $\mu^* = {}^2/{}_3 \rho^*/c^2$ (therefore the difference $-p^* = \rho^*/3$ to the full energy density ρ^* is 'dark').

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The apparent luminosity turns out to be

$$I(z) = \frac{LH^2}{4\pi c^2} [(1+z)\ln(1+z)]^{-2}, \quad (7)$$

where $L \equiv 4\pi I d_L^2$ is the absolute luminosity of a radiation source. To compare this result with the SNe-Ia data and the CCM-prediction later on, it may be converted to the SUM distance modulus

$$m_{\text{SUM}} - M = 5 \log[(1+z)\ln(1+z)] + 25 + 5 \log\left(\frac{c/H}{\text{Mpc}}\right) . \quad (8)$$

Since the redshift parameters z are independent of time for sources at rest in universal ('comoving') coordinates, so are the magnitudes and all other quantities, too, which are functions of z .

The intrinsic limitations of proper length and proper time

The intervals (dt_{SRT} , dl_{SRT}) of proper time and length are defined necessarily *together*, according to the line element of special relativity theory (SRT) within local inertial frames

$$d\sigma_{SRT}^2 = c^2 dt_{SRT}^2 - dl_{SRT}^2 . \quad (9)$$

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Now *integrated* coordinates (r' , $T' \equiv 1/H + t'$) implicitly given by

$$t^* \equiv \frac{\ln(HT')}{H}, \quad r^* \equiv \frac{r'}{HT}, \quad (10)$$

transform the stationary line element (1), (2) approximately into that of SRT

$$d\sigma'^2 = \left[1 - \left(\frac{r^*}{R_H} \right)^2 \right] c^2 dT'^2 + 2 \left(\frac{r^*}{R_H} \right) c dT' dr' - dr'^2 - r'^2 d\Sigma'^2 . \quad (11)$$

where $d\Sigma'$ is the element of a Euclidean spherical surface. Evidently $d\sigma'$ as SRT-approximation can only be valid within *local* cosmic areas limited to extensions $r^* < R_H$ where $R_H \equiv c/H$ is the constant Hubble radius. Since no coordinate origin is preferred, there may be many 'locally' coherent regions where the special-relativistic concepts of proper length and time only apply.

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$$T' \stackrel{!}{>} r'/c \geq 0 \quad (12)$$

that there shouldn't be any local structures older than $T_H \equiv 1/H$ with respect to their proper time. Therefore T_H has not necessarily to be the age of the universe as a whole and what is called expansion of space may the other way round be understood a universal condensation of material structures arising again and again.

Relation (11) [or (15)] allows the 'big bang' concept to apply only to regions distinctly smaller than R_H . Since with respect to the universal space no special position of the coordinate origin $r^* = 0$ is preferred, such local regions of gravitational creation may be spread all over the universe.

If without the universal coordinate r^* only the universal time t^* had been transformed according to

$$t^* \equiv \frac{\ln(HT')}{H} \quad (13)$$

this would have resulted in a Friedman-Lemaître-Robertson-Walker (FLRW) form. In particular for comparison with other models it is convenient to rewrite the stationary line element (1), (2) in such a form now

$$d\sigma_{\text{SUM-FLRW}}^2 = c^2 dT'^2 - (HT')^2 (dr^{*2} + r^{*2} d\Sigma^{*2}) \quad (14)$$

without thereby changing any physical results, of course. It is easily verified that, in particular, the Hubble relation (4) holds from (14) in its time-independent form, too!

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The FLRW-form (14) with its scale factor $a(t') \equiv HT' \equiv 1+Ht'$ is no longer without singularity. However, it is important to see from (11) that

$$r^* \stackrel{!}{<} R_H . \quad (15)$$

Therefore the integrated time T' as a quasi-Minkowskian coordinate approximation to a local proper-time integral t_{SRT} is not at all suitable to hold at or beyond cosmic distances $r^* \rightarrow c/H$. Thus the coordinate time T' of any FLRW-form cannot be a uniform proper time all over the universe since proper time is always given only together with proper length, i.e. in fact locally.

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Today the scale factor is usually fixed to the value 1 by the choice of appropriate units to take into account the temporal approximate validity of SRT in our local region of space and time. But local SRT requires this scale factor to be fixed to value 1 again and again as in e.g. another 10 billion years, whereas pure GRT would require roughly the double value then. This is only a simple example for a struggle of SRT and GRT as well as for the self-restoring validity of local SRT. Without this self-restoring aspect, SRT could not be valid in freely falling local inertial frames, since the Lorentz transformation is not integrable as Einstein himself has shown yet before 1915.

The universal embedding of conventional GRT

Going on from the homogeneous stationary large scale background to cover local inhomogeneities, too, there might apply an embedded line element of General Relativity Theory (GRT) which is

$$d\sigma^{*2} = \zeta^{*2} d\sigma_{\text{GRT}}^{*2} \quad (16)$$

where $d\sigma^{*2}_{\text{GRT}}$ is Einstein's solution determined by the vacuum equations $R_{ik} = 0$. In addition, now

$$\zeta^* = e^{H(t^*, \vec{r}^*) t^*} \quad (17)$$

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$$H \equiv \overline{H(t^*, \vec{r}^*)} \stackrel{!}{=} \text{constant} \Big|_{H\Delta t^*, H|\Delta \vec{r}^*|/c > X} \quad (18)$$

if averaged over universal scales of space and time sufficiently large, what should mean $0.1 < X \leq 1$.

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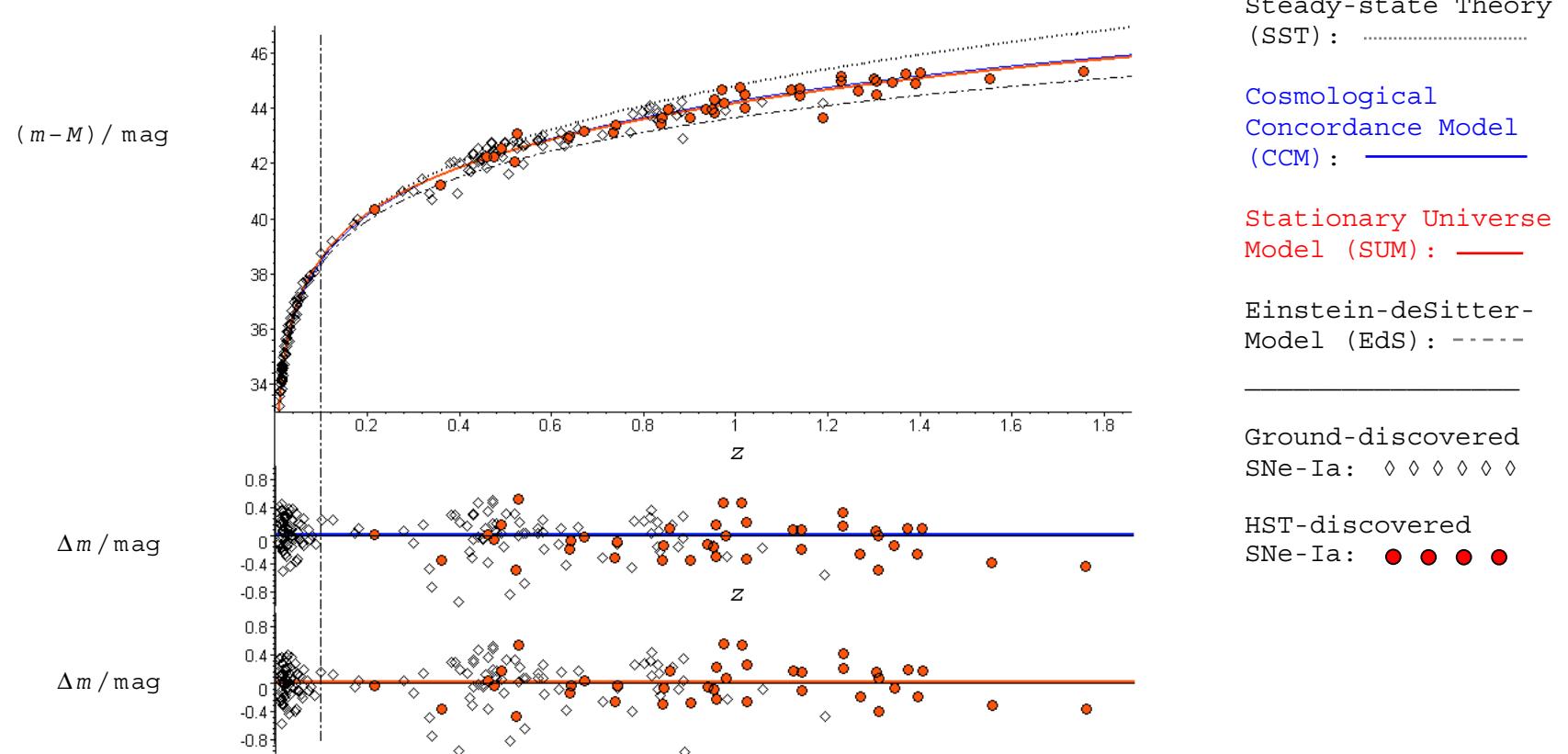
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In this case the equations (4) and (8) for redshift and distance modulus stay essentially the same except for a substitution of H by

$$H_E \equiv H \left(t_0^* - \frac{|\vec{r}_E^* - \vec{r}_0^*|}{c}, \vec{r}_E^* \right) \quad (19)$$

where (t_0^*, \vec{r}_0^*) mean time and place of observation while 'E' indicates 'emission' or more general an 'event'. This generalization allows to take into account local cosmic inhomogeneities. This proves important in what may follow finally.

Confrontation with the Supernovae-Ia data



Comparing the SUM magnitude-redshift prediction (8) with the SNe-Ia data of Riess et al. [2], [3] and the CCM, I want to point out to the straightforward agreement in the high-redshift range $0.1 < z < 2$ where the universe may be regarded homogeneous and isotropic.

For both SUM and CCM respectively a Hubble contrast reported by Jha, Riess, Kirshner [4] is taken into account at the best possible value in the low-redshift range on the left of the broken line.

Though the panels above show clear indication of a stationary large scale universe the SUM presented here is capable of embedding the whole CCM-cosmos, too, if necessary.

Concluding remark

Since the lifetimes of stars, galaxies and progenitor structures are finite of course, in view of stationarity there must happen new formation of those structures from time to time - everywhere in the universe, again and again - according to a principle of stationary cosmology, what apparently violates the law of entropy if unrestricted, though:

Given a stationary universe, all material components are determined by the requirement that they are gravitationally recreated according to the laws of quantum physics at the same rates as they have disappeared before in extreme gravitational centers, growing to cores of hot originative 'local bang' events.

It means, the material components of a stationary universe must exist at approximately those rates which are calculated from the 'big bang' model, actually.

The stationarity of this SUM evident from its redshift parameters is supported by the self restoring validity of SRT within 'local' inertial frames. Though, this stationarity is not a 'steady state' but a lively process!

The original concept and development of the stationary model up to the first conclusion of a local Hubble contrast is documented in this book [1].

If you want to check statements, figures, or equations of this talk, you can find the pdf-sheets at <independent-research.org> including the SNe-Ia data compilation of Riess et al. (2004/07) exactly as used here.

Thank you for your attention.

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Literature

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- [2] RIESS A. G. et al.: TYPE Ia SUPERNOVA DISCOVERIES AT $z > 1$ FROM THE HUBBLE SPACE TELESCOPE ..., ApJ **607** (2004) 665-687
- [3] RIESS A. G. et al.: New Hubble Space Telescope Discoveries of Type Ia Supernovae at $z=1$..., ApJ **659** (2007) 98-121
- [4] JHA S., RIESS A. G., KIRSHNER R. P.: Improved Distances to Type Ia Supernovae ..., ApJ **659** (2007) 122-148
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Appendix

Meanwhile you may ask how this model of unique simplicity, and in particular its clear stationarity with redshift parameters independent of time, could happen to escape its discovery in those times, when the so-called Steady-state Theory was developed and widely discussed? Three important reasons may be:

- a) the coordinate time t' of the FLRW form has been misunderstood as a universal proper time whereas this concept of proper time only applies locally.
- β) a negative gravitational pressure $p^* = -^1/{}_3\rho^* < 0$ has not been considered a physical option before the breakthrough of the SNe-Ia observations (though really plausible, s. above), and
- γ) the time-independence of redshift (4) with its significant Hubble constant H ($\equiv H_s = da/dt'$ where $a = HT'$ the SUM scale factor) might have been concealed by the rather misleading conventional Hubble parameter $H_c \equiv H_p(T') \equiv (da/dt')/a = 1/T'$, unfortunately indicating a dependence of time where no such dependence exists.

Naturally, the stationary 'deceleration' parameter is $q_{\text{SUM}} = 0$ which value has been interpreted - I got to know about later - as a 'coasting' cosmology [5] in a completely other context without stating the decisive stationarity of redshift or the postulate of spatial flatness due to a constant universal speed of light $c^* = c$ nor the universal scalar form of (1), (2) corresponding to a stationary embedding of SRT or other essential features of the approach presented here.

Some more material for answering questions

- 1) There may be some relationship between a the negative gravitational pressure and a local decrease of entropy since *increasing* entropy of a gas is clearly associated to its *positive* pressure.
- 2) It's simply impossible to work out high precision cosmology without essential priors ...
- 3) I think it's legitimate to claim the Supernovae Ia magnitude-redshift data to represent the most valuable cosmological breakthrough of the last decade because their confrontation with competing theories requires the least input of unproven hypotheses about the universe.
- 4) At least one thing I know for sure: If you were satisfied with today's Concordance Model, you wouldn't be here in this session.

The CCM scale factor in view of the SUM

Neglecting radiation in this context and setting the phenomenological pressure of matter $p_M = 0$, Einstein's equations extended by his cosmological term Λg_{ik} , yield the CCM scale factor

$$a_{\text{CCM}}(t') = \left\{ \left(\frac{1}{\Omega_\Lambda} - 1 \right) \sinh^2 \left[\frac{1}{2} \ln \left(\frac{1-\sqrt{\Omega_\Lambda}}{1+\sqrt{\Omega_\Lambda}} \right) - \frac{3}{2} \sqrt{\Omega_\Lambda} H t' \right] \right\}^{1/3} \quad (19)$$

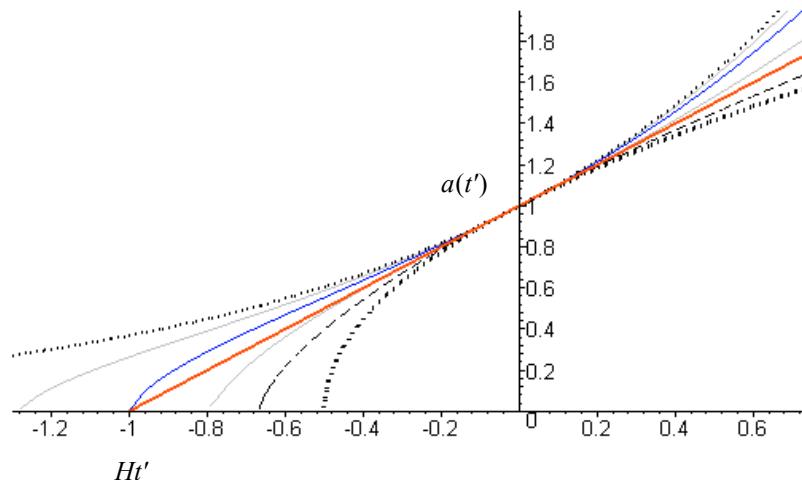
where t' is the FLRW coordinate time, $\Omega_\Lambda \equiv \rho_\Lambda / \rho_c$ with ρ_c the critical density and $\rho_M + \rho_\Lambda = \rho_c$ for a spatially Euclidean model.

Steady-state Theory
(SST) :

Cosmological
Concordance Model
(CCM) :
— $\Omega_\Lambda = 0.90$
— $\Omega_\Lambda = 0.73$
— $\Omega_\Lambda = 0.40$

Stationary Universe
Model (SUM) : —

Einstein-deSitter-Model
(EdS) : - - -



In contrast to other values (grey solid lines), the best-fit CCM parameter $\Omega_\Lambda \approx 0.73$ (blue line) seems determined by the condition [6] that it should meet the SUM scale factor (red straight line) at its 'boundaries', i.e. at $Ht' = -1$ and at $Ht' = 0$.

In view of the standard 'big bang' model an inflation scenario is certainly needed to arrive with e.g. an effectively flat universe, 'superhorizon' scales, and probably all those features one simply would start from, given a stationary universe according to SUM.